MEA 712: (An Introduction to) Mesoscale Atmospheric Modeling
Fifth computing assignment
Due: Start of class on Tuesday 15 September
This assignment will be turned in for a grade

We will now consider the performance of our upstream advection scheme for a variety of conditions. This is meant to build on what you have done in the first four mini computing assignments. You should use your final code from assignment 4 as your starting point, including the periodic lateral boundary condition.

Consider the following equation for constant advection:

$$\frac{\partial \psi}{\partial t} = -c\frac{\partial \psi}{\partial x}. \tag{1}$$

The true/analytic solution for constant advection would be perfect amplitude preservation (neither damping nor amplification) and perfect phase speed conservation (the waves move at exactly the speed c). We will use your upstream scheme (forward in time, backward in space) code from assignment 4, but with a varying initial condition and current speed, as explained below.

1. Perform the following 16 model realizations. Keep Δt and Δx fixed to their original settings from the prior assignments (10.0 s and 100.0 m, respectively). You should run each realization forward for 10 timesteps (i.e. a total of 100.0 s).

a) Start the model with each of the following initial conditions: a cosine wave with wavelengths of 2Δx, 4Δx, 10Δx, and 20Δx. In other words, your initial condition should be

$$\psi(t=0) = \cos\left(\frac{2\pi x}{n\Delta x}\right), \tag{2}$$

where n=2, 4, 10, and 20.

b) For *each* wavelength, run the model with varying choices of c such that you are able to test the following parameter space: μ=0.1, 0.5, 1.0, and 1.5, where

$$\mu \equiv c\frac{\Delta t}{\Delta x}. \tag{3}$$

This parameter, μ, is called the "Courant number". It turns out to be relevant to the stability and performance of many numerical schemes, as we shall see throughout this class.

c) Carefully analyze your results from the 16 model runs. You should inspect *each* run's output at *all* output times to be sure you are seeing the whole picture.

Note: It is *expected* that some of your runs will "blow up" (the values will go to $\pm\infty$, which in FORTRAN is to say `NAN`). But, only a *few* of them!

2. Comment on the **speed preservation** of the upstream scheme (recalling that the true analytic solution is translation of the waveform at speed c).

a) In what way (if at all) is speed preservation a function of the wavelength of the feature?

b) In what way (if at all) is speed preservation a function of the Courtant number, μ?

c) Considering the logic of the numerical scheme, interpret these properties you have discovered in a and b. Why does the upstream scheme behave this way?

(Continued)

3. Comment on the **amplitude preservation** of the upstream scheme (recalling that the true analytic solution is translation of the waveform with neither damping nor amplification).

a) In what way (if at all) is amplitude preservation a function of the wavelength of the feature?

b) In what way (if at all) is amplitude preservation a function of the Courtant number, μ?

c) Considering the logic of the numerical scheme, interpret these properties you have discovered in a and b. Why does the upstream scheme behave this way?

4. Turn in for credit:

a) One plot each of the final time (i.e. after 10 timesteps) from the following 4 runs: 2Δx wave with μ=1.0; 4Δx wave with μ=0.1; 10Δx wave with μ=0.5; 20Δx wave with μ=1.5. If any of your solutions are unstable (go to `NAN`), then show the time immediately before all values are `NAN` (i.e. show the final time with finite values, however ugly!). **Be sure each plot is labelled appropriately (including the run name and the timestep at which the plot is valid).**

b) Your written responses to questions 2 and 3.